SYNOPSIS

Proposed Title: Non-linear waves of plasma in the non-planar geometry

Submitted by

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Registration No:VB-2355 of 2016-17
Date of Registration for Ph.D. : 28.11.2016

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Keywords: KdV equation,KdV-Burger equation, ZK equation,ZKB equation ,Planar and Non-planar geometry,Solitons and Shock waves.

The plasma is an ionized gas ensemble of charged particles, capable of exhibiting collective behavior.The properties of cylindrical and spherical ion acoustic solitary waves are investigated in a three component unmagnetized,collisionless plasma consisting of worm ion fluid and superthermally electrons and positrons in a non-planar cylindrical and spherical geometries.Using Reductive Perturbation Technique(RPT)the planar and non-planar KdV equation,Burger equation and KdV-Burger equation are derived.
The KdV equation is

$$\frac{\partial \phi}{\partial t} + A\phi\frac{\partial \phi}{\partial x} + B\frac{\partial^3 \phi}{\partial x^3} = 0 \tag{1}$$

The Burger Equation is

$$\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} = C\frac{\partial^2 \phi}{\partial \xi^2} \tag{2}$$

and the KdV-Burger equation is

$$\frac{\partial \phi}{\partial \tau} + \frac{\nu}{2\tau}\phi + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} + C\frac{\partial^2 \phi}{\partial \xi^2} = 0 \tag{3}$$

when $\nu = 0$ the equation represents the equation in the planar geometry and for $\nu = 1, 2$ the equation represents in the Cylindrical and Spherical co-ordinate geometry.
The solution of KdV equation represent soliton , The solution of the Burger equation represents shock waves and the solution of the KdV-Burger equation represents soliton as well as shock waves. Previously many researchers solved the equations numerically in different ways and made comparison with analytical and numerical solutions.
The study of solitions and shock waves in plasma is a rapidly growing research field. Very few works have been done on plasma in non-planar geometry. Recently, in 2014 Dr. Pankaj Kr Mandal worked on 'Computational study on non-linear structure in plasma'. The effects of several parameters on the properties of ion-acoustic two-soliton systems in quantum electron–positron–ion plasmas have been discussed in planar and non-planar geometries. It has been shown that the properties of ion-acoustic two-soliton systems are affected significantly due to cylindrical and spherical geometries. The amplitude of the cylindrical two-soliton system is smaller than that of the spherical two-soliton system for small values of τ .
The propagation of ion-acoustic two-soliton systems is quite different from the propagation of ion-acoustic two-soliton systems in a nonplanar geometry. In a bounded cylindrical geometry Zakharov–Kuznestov– Burger (ZKB) equation has been derived by them, for the first time, using reductive perturbation technic. The ZKB equation is

$$\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} + C\frac{\partial}{\partial \xi}\left(\frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial^2 \phi}{\partial \zeta^2}\right) - D\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\partial^2 \phi}{\partial \zeta^2}\right) = 0 \tag{4}$$

The KPB equation in spherical and cylindrical geometry can be obtained by applying the stretch co-ordinate

$xi = \epsilon^{\frac{1}{2}}(r - v_0 t)$,$\chi = \epsilon^{\frac{-1}{2}}$,$\tau = \epsilon^{\frac{1}{2}} t$, where ϵ is small parameter.

$$\frac{\partial}{\partial \zeta}\left(\frac{\partial \phi}{\partial \tau} + A\phi\frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} - C\frac{\partial^2 \phi}{\partial \zeta^2} + \frac{\nu}{2\tau}\phi\right) + \frac{D}{\tau^{2\Xi}}\left(\frac{\partial^2 \phi}{\partial \chi^2} + \frac{\Xi(\nu-1)}{\chi}\frac{\partial \phi}{\partial \chi}\right) = 0 \qquad (5)$$

where $\Xi = 0, 1$ for planar and non-planar geometries respectively. A and C are the co-efficient of nonlinearity and dissipation respectively,whereas B and D are the co-efficient of dispersion.$\nu = 0$ and $\Xi = 0$ corresponds to the KBP equation in the planar geometry,whereas $\nu = 1, 2$ and $\Xi = 1$ correspond to the KBP equation in cylindrical and spherical geometries respectively.

System of coupled nonlinear ordinary differential equations has been derived from ZKB equation and has been solved numerically using fourth order Runge–kutta method. Equilibrium points and their features has been studied dynamically in the neighbourhood of these points. With the variation of the non-extensive parameter and the kinematic viscosity parameter some important features in the nonlinear waves like oscillatory shocks to steady state propagation and vis-a-vis steady state propagation to oscillatory shocks emerge. When the values of the non-extensive parameter decrease, the phase portrait of the system shows that the change from stable spiral to stable closed and stable to unstable equilibrium happens . When the effect of dissipative term i.e. kinematic viscosity has been considered some other significant features also evolve. Now I am trying to solve the KdV ,KdV Burger equation, ZK equation,ZKB equation in cylindrical and Spherical Geometry numerically and to observe the effects of solitons and shock waves for different values of the parameters in the non-planar geometry using MATLAB progamming.

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